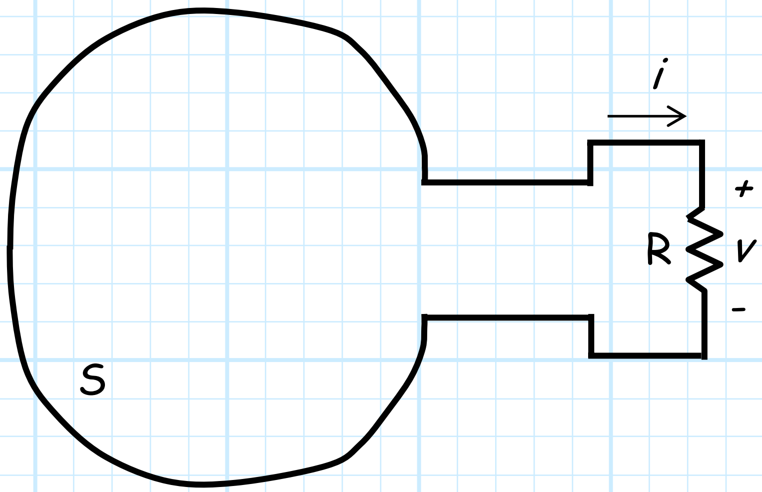
The Electromotive Force

Consider a wire loop with surface area S, connected to a single **resistor** R.

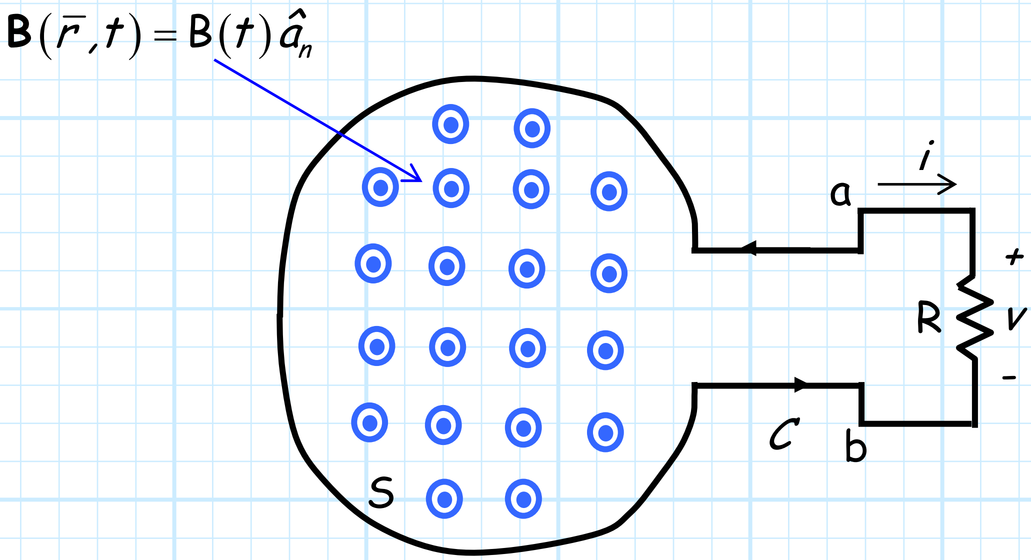


Since there is **no** voltage or current **source** in this circuit, both voltage v and current i are **zero**.

Now consider the case where there is a **time-varying** magnetic flux density $\mathbf{B}(\bar{r},t)$ within the loop only. In other words, the magnetic flux density outside the loop is zero (i.e., $\mathbf{B}(\bar{r},t)=0$ outside of S).

Say that this magnetic flux density is a **constant** with respect to position, and points in the direction normal to the surface S. In other words;

$$\mathbf{B}(\bar{r},t)=B(t)\ \hat{a}_n$$



According to **Faraday's Law**:

$$\oint_C \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = -\frac{\partial}{\partial t} \iint_S \mathbf{B}(\bar{r},t) \cdot \overline{ds}$$

$$\oint_C \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = -\frac{\partial}{\partial t} \iint_S B(t)\,\hat{a}_n \cdot \overline{ds}$$

$$\int_a^b \mathbf{E}(\bar{r}) \cdot \overline{d\ell} + \int_b^a \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = -\frac{\partial B(t)}{\partial t} \iint_S ds$$

The contour from point a to point b is along a wire, which we presume to be a **perfect conductor**. Since the electric field within a perfect conductor is equal to **zero**, we find:

$$\int_a^b \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = 0$$

Likewise, if we integrate through the **resistor** from point *b* to point *a*, we find:

$$\int_b^a \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = -\int_a^b \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = -v$$

Finally, we note that:

$$\iint_S ds = S$$

where S is the **surface area** of the loop.

Combining these results, we find:

$$v = S \frac{\partial B(t)}{\partial t}$$

Or, recalling that **magnetic flux** Φ is defined as:

$$\iint_S \mathbf{B}(\bar{r},t) \cdot \overline{ds} = \Phi(t)$$

we can write:

$$v = S \frac{\partial B(t)}{\partial t} = \frac{\partial \Phi(t)}{\partial t}$$

For this case, the **voltage** across the resistor is proportional to the **time derivative** of the **total magnetic flux** passing through the aperture formed by contour C.

Using the circuit form of Ohm's Law, we likewise find that the current in the circuit is:

$$\begin{aligned} i &= \frac{V}{R} \\ &= \frac{S}{R} \frac{\partial B(t)}{\partial t} \\ &= \frac{1}{R} \frac{\partial \Phi(t)}{\partial t} \end{aligned}$$

In other words, time-varying magnetic flux density can **induce** a voltage and current in a circuit, even though there are **no** voltage or current **sources** present!

The voltage created is known as the **electromotive force.**

The electromotive force is the basic **phenomenon** behind the behavior of:

1. Electric power **generators**
2. Transformers
3. Inductors